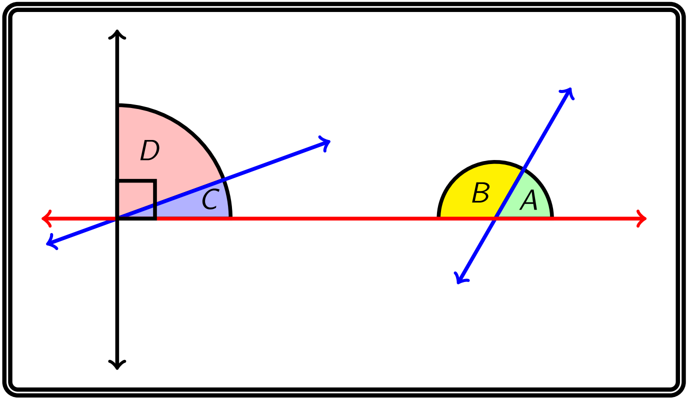
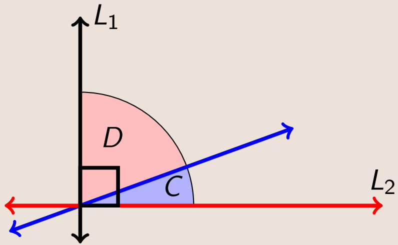
Trigonometry DLA Series



Complementary & Supplementary Angles

In this DLA, we are are going to look at angles that have a sum of 90° and 180°.

When two angles have a sum of 90°, they are called **Complementary Angles**.

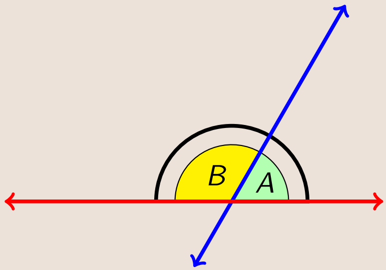


When we assume that $L_1 \perp L_2$, they form a 90° angle. Therefore

$$m\angle C + m\angle D = 90^\circ.$$

Angles C and D are called complementary angles.

When two angles have a sum of 180°, they are called **Supplementary Angles**.



When we split a straight angle which has a measure of 180° into two angles A and B,

therefore
$m\angle A + m\angle B = 180^\circ$.

Angles A and B are called supplementary angles.

When two angles are **Complementary Angles**, they are **Complement** of each other.

When two angles are **Supplementary Angles**, they are **Supplement** of each other.

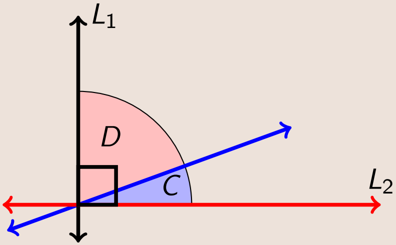
Type	First Angle	Second Angle
Complementary Angles	x°	$(90 - x)^\circ$
Supplementary Angles	x°	$(180 - x)^\circ$

Example:

Find two complementary angles such that one of them is 20° more than its complement.

Solution:

Let x be the measure of one of the angles, then its complement has to be $90 - x$.



When we assume that $L_1 \perp L_2$, they form a 90° angle.

$$m\angle C = x^\circ,$$
$$m\angle D = (90 - x)^\circ,$$

$$m\angle D = m\angle C + 20^\circ$$

Solution(continued):

$$\boxed{m\angle D} = \boxed{m\angle C} + 20^\circ \qquad \text{(Given Information)}$$

$$\boxed{90 - x} = \boxed{x} + 20 \qquad \text{(Substitution)}$$

$$\begin{aligned}
90 - x - x - 90 &= x + 20 - x - 90 && \text{(Subtraction Property)}\\
-2x + 0 &= -70 + 0 && \text{(Inverse \& Simplify)}\\
-2x &= -70 && \text{(Identity)}\\
x &= 35 && \text{(Division Property)}
\end{aligned}$$

So the angle is 35°, and its complement is $90 - {\color{red}35} = 55^\circ$.

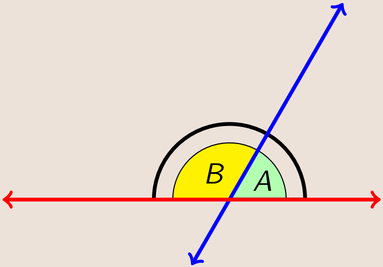
$$\boxed{35^\circ \text{ and } 55^\circ}$$

Example:

Find two supplementary angles such that one of them is 30° less than 4 times its supplement.

Solution:

Let x be the measure of one of the angles, then its supplement has to be $180 - x$.



$m\angle A = x^\circ$
$m\angle B = (180 - x)^\circ$

$m\angle A = 4 \cdot m\angle B - 30$

Solution(continued):

$$\begin{aligned}
\boxed{m\angle A} &= \boxed{4 \cdot m\angle C} - 30^\circ && \text{(Given Information)} \\
\boxed{x} &= \boxed{4(180 - x)} - 30 && \text{(Substitution)} \\
x &= 720 - 4x - 30 && \text{(Distribution Property)} \\
x &= 690 - 4x && \text{(Simplify)} \\
x + 4x &= 690 - 4x + 4x && \text{(Addition Property)} \\
5x &= 690 && \text{(Inverse \& Simplify)} \\
x &= 138 && \text{(Division Property)}
\end{aligned}$$

So the angle is 138°, and its supplement is $180 - 138 = 42^\circ$.

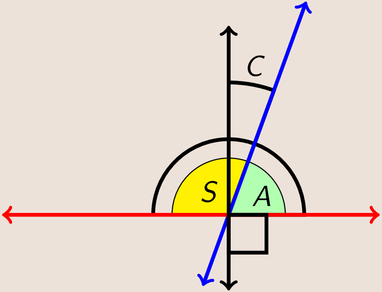
$$\boxed{42^\circ \text{ and } 138^\circ}$$

Example:

Find the measure of an angle such that the sum of its complement and its supplement is 130°.

Solution:

Let x be the measure of one of the angles, then its supplement has to be $180 - x$.



$m\angle A = x^\circ$
$m\angle C = (90 - x)^\circ$
$m\angle S = (180 - x)^\circ$

$m\angle C + m\angle S = 130^\circ$

Solution(continued):

$$\boxed{m\angle C} + \boxed{m\angle S} = 130^\circ \qquad \text{(Given Information)}$$

$$\boxed{90 - x} + \boxed{180 - x} = 130 \qquad \text{(Substitution)}$$

$$270 - 2x = 130 \qquad \text{(Simplify)}$$

$$270 - 2x - 270 = 130 - 270 \qquad \text{(Subtraction Property)}$$

$$-2x + 0 = -140 \qquad \text{(Inverse \& Simplify)}$$

$$-2x = -140 \qquad \text{(Identity)}$$

$$x = 70 \qquad \text{(Division Property)}$$

So the angle is 70°.

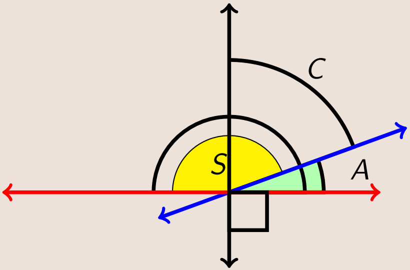
$\boxed{\text{The angle is } 70^\circ}$

Example:

Find the measure of an angle such that the difference of twice its supplement and three times its complement is 110°.

Solution:

Let x be the measure of one of the angles, then its supplement has to be $180 - x$.



$m\angle A = x^\circ$
$m\angle C = (90 - x)^\circ$
$m\angle S = (180 - x)^\circ$

$2 \cdot m\angle S - 3 \cdot m\angle C = 110^\circ$

Solution(continued):

$$\boxed{2 \cdot m\angle S} - \boxed{3 \cdot m\angle C} = 110^\circ \qquad \text{(Given Information)}$$

$$\boxed{2(180 - x)} - \boxed{3(90 - x)} = 110 \qquad \text{(Substitution)}$$

$$\begin{aligned}
360 - 2x - 270 + 3x &= 110 && \text{(Distibution Propoerty)}\\
x + 90 &= 110 && \text{(Simplify)}\\
x + 90 - 90 &= 110 - 90 && \text{(Subtraction Property)}\\
x + 0 &= 20 && \text{(Inverse \& Simplify)}\\
x &= 20 && \text{(Identity)}
\end{aligned}$$

So the angle is 20°.

$$\boxed{\text{The angle is } 20^\circ}$$

Start at ELAC, Go Anywhere